**University of Insubria**

DIPARTIMENTO DI SCIENZA E ALTA TECNOLOGIA
PhD in Computer Science and Mathematics of Calculus

# PhD Second Year Report

PhD Student: *M. Tarsia**    Tutor: *D. Cassani*    Advisor: *E. Mastrogiacomo*

Academic Year 2019–2020
14/10/2020

## Training Activity

- Seminary: *Learning and Filtering in an Uncertain Setting*, S. N. Cohen (Univ. of Oxford). 2 October 2019, DiECO, Varese.
- Seminary: *Adaptive iterative regularization in variable exponent Lebesgue spaces*, C. Estatico (Univ. of Genova). 21 November 2019, DiSAT, Como.
- RISM event: *L'intreccio tra Matematica e Filosofia: occasioni o tentazioni?*. 28 - 29 November 2019, C. Cattaneo College, Varese. Web page: `here`.
- Seminary: *A Multigrid Perspective on the Parallel Full Approximation Scheme in Space and Time*, M. Bolten (Bergische Univ. of Wuppertal). 4 December 2019, DiSAT, Como.
- Seminary: *Sparse Linear Models for Recommender Systems*, P. Cremonesi (Milan Polytechnic). 10 December 2019, DiSAT, Como.
- Seminary: *The Alternating Direction Multipliers Method for constrained optimization*, A. Buccini (Univ. of Cagliari). 7 January 2020, DiSAT, Como.
- Course: *Set-valued convex analysis*, Ç. Ararat (Bilkent Univ.). 20, 22, 24 and 27 January 2020, DiECO, Varese.
- Course: *Nonlinear and nonstationary signal decomposition and analysis. Theoretical and numerical aspects and applications*, A. Cicone (Univ. of Insubria). 17 - 21 February 2020, DiSAT, Como.
- Course: *Topics from the Gaussian World*, F. Caravenna e M. Rossi (Univ. of Milano-Bicocca). 19 and 25 March, 2, 8, 16, 23 and 29 April, 6, 13, 20 and 27 May 2020, online meetings (Zoom). Web page: `here`.
- Passed exams (see my PhD First Year Report):

  √ *An Introduction to Regularity Structures*, F. Caravenna.

  √ *(Markov Chain) Monte Carlo Simulation*, A. Mira; including research (see below).

  Reports written in LaTeX available on `my web page`.

  √ B2 First, Cambridge English (my final results will be officially available in a few months).

  [⇒ Please note: the exams are all over!]

## Tutoring Activity

► Courses of Study in Mathematics and Physics: *Analisi Matematica 2*, E. Casini (Univ. of Insubria), DiSAT, Como. 8 May (10:30-13:00), 5 June (10:30-13:00), 19 June (14:30-17:00), 10 July (9:30-12:00) 2020, online meetings (Microsoft Teams).

Notes written in LaTeX available on `my web page`.

Handwritten notes and streaming videos available for the channel in Microsoft Teams of the course.

► Course of Study in Mathematics: *Probabilità e Statistica*, A. Posilicano (Univ. of Insubria), DiSAT, Como. 12 May (16:00-18:30), 18 May (14:00-16:30), 25 May (14:00-16:30), 1 June (14:00-16:30), 9 June (11:00-13:30), 12 June (14:00-16:30) 2020, online meetings (Microsoft Teams).

Handwritten notes and streaming videos available for the channel in Microsoft Teams of the course.

*E-mail: `mtarsia1@uninsubria.it`. Web page: `https://www.uninsubria.it/hpp/marco.tarsia`.

**Supervised exams** (Microsoft Teams)**.**

- Corso di Studio in Chimica e Chimica Industriale: *Matematica 1*, A.G. Setti (Univ. of Insubria), DiSAT, Como. 26 June 2020 (9:30-12:00).
- Corso di Studio in Chimica e Chimica Industriale: *Matematica 1*, A.G. Setti (Univ. of Insubria), DiSAT, Como. 13 July 2020 (9:00-11:30).
- Corso di Studio in Matematica e Fisica: *Analisi Matematica 2*, E. Casini (Univ. of Insubria), DiSAT, Como. 1 September 2020 (14:15-16:45).
- Corso di Studio in Matematica e Fisica: *Analisi Matematica 2*, E. Casini (Univ. of Insubria), DiSAT, Como. 15 September 2020 (14:15-16:45).

**Evaluated exams.**

- Corso di Studio in Ingegneria Ambientale e per la Sostenibilità degli Ambienti di Lavoro: *Metodi Matematici e Statistici per l'Ingegneria*, A. Mira (Univ. of Italian Switzerland). Eleven term papers, September 2020.

## Research Activity

★ PhD thesis: *Time-consistent optimal controls in risk constrained portfolio management* in collaboration with E. Mastrogiacomo (Univ. of Insubria).

Our basic idea is to produce a theory of stochastic optimal controls, at least up to an existence theorem and a maximum principle regarding Hamiltonian systems (necessary and sufficient conditions for optimality), which works in dealing with constraints imposed on control or on the state trajectory - issues possibly close to each other for controls in feedback form - and all that to model and describe a portfolio selection problem with constraints possibly characterized through scalar or dynamic (conditional) risk measures in such a way that, employing a notion of optimality or equilibrium more general than the classical one, a time-consistency property of the decision-maker is guaranteed, which makes the optimal policy relevant (implementable) in practice. The cost functional should contain indeed a utility function but also a discount function which would be not exponential.

Ultimately, we expect to have to change the adjoint equations and probably the boundary conditions of the adjoint processes (thus BSDEs), so eventually the Hamiltonian functions themselves.

The main starting reference is therefore the classical "J. Yong, X.Y. Zhou. *Stochastic Controls. Hamiltonian Systems and HJB Equations.* Springer, Heidelberg (1999)." where, by the way, is considered the possibility of pointwise constraints, which is almost not developed (especially about the control), whereas are studied state constraints that match a condition on the cost functional (in relation to which we find the so-called transversality condition) and moreover something about reachable sets.

We present below a summary of the existing literature on that subject, starting from the research closest to our purpose. We remark that one of our beliefs is that an in-depth study of risk measures turns out to be useful, if not crucial, for our main goal through some of the following fields: risk measures from BSDEs; law invariance; comonotonicity; less than convexity; links with the underlying scenario space; consistency with respect to appropriate orderings; connections with game theory; set-valued case (all that not just for random variables or portfolio vectors which are bounded and also to get robust representation formulas, hoping finally for the chance of a reasonable limit in the conditional framework).

**References on our purpose.**


- N. El Karoui, S. Peng, M.C. Quenez. *A dynamic maximum principle for the optimization of recursive utilities under constraints.* Annals of applied probability (2001).
- Y. Hu, P. Imkeller, M. Müller. *Utility maximization in incomplete markets.* The Annals of Applied Probability (2005).
- S. Ji, X.Y. Zhou. *A maximum principle for stochastic optimal control with terminal state constraints, and its applications.* Communications in Information & Systems (2006).
- T.A. Pirvu. *Portfolio optimization under the Value-at-Risk constraint.* Quantitative Finance (2007), Taylor & Francis.
- A. Debussche, M. Fuhrman, G. Tessitore. *Optimal control of a stochastic heat equation with boundary-noise and boundary-control.* ESAIM: Control, Optimisation and Calculus of Variations (2007).
- I. Ekeland, T.A. Pirvu. *Investment and consumption without commitment.* Mathematics and Financial Economics (2008), Springer.
- P. Imkeller, G. Dos Reis. *Path regularity and explicit convergence rate for BSDE with truncated quadratic growth.* Stochastic Processes and their Applications (2010), Elsevier.

- P. Cheridito, Y. Hu. *Optimal consumption and investment in incomplete markets with general constraints.* Stochastics and Dynamics (2011), World Scientific.
- I. Ekeland, O. Mbodji, T.A. Pirvu. *Time-Consistent Portfolio Management.* SIAM Journal on Financial Mathematics (2012).
- S. Moreno-Bromberg, T.A. Pirvu, A. Réveillac. *CRRA utility maximization under dynamic risk constraints.* Communications on Stochastic Analysis (2013).
- T. Björk, A. Murgoci, X.Y. Zhou. *Mean-variance portfolio optimization with state-dependent risk aversion.* Mathematical Finance (2014).
- K.K. Aase. *Recursive utility using the stochastic maximum principle.* Quantitative Economics (2016).
- M. Hu. *Stochastic global maximum principle for optimization with recursive utilities.* Probability, Uncertainty and Quantitative Risk (2017).
- Q. Wei, J. Yong, Z. Yu. *Time-inconsistent recursive stochastic optimal control problems.* SIAM Journal on Control and Optimization (2017).
- Y. Zhuo. *Maximum Principle of Optimal Stochastic Control with Terminal State Constraint and Its Application in Finance.* Journal of Systems Science and Complexity (2018), Springer.
- H. Frankowska, H. Zhang, X. Zhang. *Necessary optimality conditions for local minimizers of stochastic optimal control problems with state constraints.* Transactions of the American Mathematical Society (2019).


**References on risk measures.**


- F. Delbaen. *Coherent risk measures.* Cattedra Galileiana, Classe di Scienze, Pisa, Scuola Normale Superiore (2000).
- S. Kusuoka. *On law invariant coherent risk measures.* Advances in mathematical economics (2001).
- H. Föllmer, A. Schied. *Convex measures of risk and trading constraints.* Finance and Stochastics (2002), Springer.
- E. Jouini, M. Meddeb, N. Touzi. *Vector-valued coherent risk measures.* Finance and Stochastics (2004), Springer.
- K. Detlefsen, G. Scandolo. *Conditional and dynamic convex risk measures.* Finance and Stochastics (2005), Springer.
- E. Rosazza-Gianin. *Risk measures via g-expectations.* Insurance: Mathematics and Economics (2006).
- L. Rüschendorf. *Law invariant convex risk measures for portfolio vectors.* Statistics & Decisions (2006).
- H. Föllmer, I. Penner. *Convex risk measures and the dynamics of their penalty functions.* Statistics & Decisions (2006).
- C. Burgert, L. Rüschendorf. *Consistent risk measures for portfolio vectors.* Insurance: Mathematics and Economics (2006).
- U. Horst, S. Moreno-Bromberg. *Risk minimization and optimal derivative design in a principal agent game.* Mathematics and Financial Economics (2008), Springer.
- F. Delbaen, S. Peng, E. Rosazza Gianin. *Representation of the penalty term of dynamic concave utilities.* Finance and Stochastics (2010), Springer.
- A.H. Hamel, F. Heyde. *Duality for set-valued measures of risk.* SIAM Journal on Financial Mathematics (2010).
- I. Ekeland, W. Schachermayer. *Law invariant risk measures on* $L^\infty(\mathbb{R}^d)$. Statistics & Risk Modeling (2011).
- U. Horst, S. Moreno-Bromberg. *Efficiency and equilibria in games of optimal derivative design.* Mathematics and Financial Economics (2011), Springer.
- I. Ekeland, A. Galichon, M. Henry. *Comonotonic measures of multivariate risks.* Mathematical Finance (2012).
- I. Penner, A. Réveillac. *Risk measures for processes and BSDEs.* Finance and Stochastics (2015), Springer.


**References on the fundamentals.**


- V. Strassen. *The existence of probability measures with given marginals.* The Annals of Mathematical Statistics (1965).
- R.T. Rockafellar. *Convex analysis.* Princeton University Press (1970).
- I. Ekeland. *On the variational principle.* Journal of Mathematical Analysis and Applications (1974).
- K.M. Chong. *Some extensions of a theorem of Hardy, Littlewood and Pólya and their applications.* Canadian Journal of Mathematics (1974).

- S.E. Nevius, F. Proschan, J. Sethuraman. *Schur functions in statistics. II. Stochastic majorization.* The Annals of Statistics (1977).
- L. Rüschendorf. *Ordering of distributions and rearrangement of functions.* The Annals of Probability (1981).
- S. Peng. *A general stochastic maximum principle for optimal control problems.* SIAM Journal on control and optimization (1990), SIAM.
- S. Peng. *Backward stochastic differential equations and applications to optimal control.* Applied Mathematics and Optimization (1993).
- F. Delbaen, W. Schachermayer. *A general version of the fundamental theorem of asset pricing.* Mathematische Annalen (1994).
- N. El Karoui, S. Peng, M.C. Quenez. *Backward stochastic differential equations in finance.* Mathematical Finance (1997).
- J. Yong, X.Y. Zhou. *Stochastic Controls. Hamiltonian Systems and HJB Equations.* Springer, Heidelberg (1999).
- S. Basak, A. Shapiro. *Value-at-risk-based risk management: optimal policies and asset prices.* The Review of Financial Studies (2001).
- I. Ekeland, E. Taflin. *A theory of bond portfolios.* The Annals of Applied Probability (2005).
- D. Lamberton, B. Lapeyre. *Introduction to Stochastic Calculus Applied to Finance.* Chapman and Hall/CRC, Financial Mathematics Series (2007).
- A. Gundel, S. Weber. *Utility maximization under a shortfall risk constraint.* Journal of Mathematical Economics (2008).
- T. Björk. *Arbitrage theory in continuous time.* Oxford University Press (2009).
- J.P. Aubin, H. Frankowska. *Set-valued analysis.* Birkhaüser-Verlag (2009).
- H. Brezis. *Functional analysis, Sobolev spaces and partial differential equations.* Universitext, Springer (2010).
- H. Föllmer, A. Schied. *Stochastic finance: an introduction in discrete time.* Stochastic Finance, de Gruyter (2011).
- A. Klenke. *Probability theory: a comprehensive course.* Universitext, Springer (2014).
- J.C. Hull. *Options, Futures, and Other Derivatives.* Pearson (2014).
- S. Drapeau, M. Kupper, E. Rosazza Gianin, L. Tangpi. *Dual representation of minimal supersolutions of convex BSDEs.* Annales de l'Institut Henri Poincaré, Probabilités et Statistiques (2016).


**Approved projects.**

⋆ Istituto Nazionale di Alta Matematica "Francesco Severi" (INdAM), Gruppo Nazionale per l'Analisi Matematica, la Probabilità e le loro Applicazioni (GNAMPA): *Lie's Symmetries Analysis of Stochastic Optimal Control Problems with Applications* in collaboration with E. Mastrogiacomo (Univ. of Insubria), S. Ugolini (Univ. of Milan), F. De Vecchi (Univ. of Bonn).

★ Other survey: *On the Mathematical Foundation of Approximate Bayesian Computation: a Robust Set for Estimating Mechanistic Network Models* in collaboration with D. Cassani (Univ. of Insubria) and A. Mira (Univ. of Italian Switzerland).

We research relations between optimal transport theory, OTT, and the innovative methodology of approximate Bayesian computation, ABC, possibly connected to relevant metrics defined on probability measures.

Those of ABC are computational methods based on Bayesian statistics and applicable to a given generative model to estimate its a posteriori distribution in case the likelihood function is intractable. The idea is therefore to simulate sets of synthetic data from the model with respect to assigned parameters and, rather than comparing prospects of these data with the corresponding observed values as typically ABC requires, to employ just a distance between a chosen distribution associated to the synthetic data and another of the observed values.

Such methods have become increasingly popular especially thanks to the various fields of applicability which go from finance to biological science, and yet an ABC methodology relying on OTT as the one we're trying to develop was born specifically with the hope of esteem mechanistic network models, i.e. models for data network growth or evolution over time, thus particularly suitable for processing dynamic data domains; but which indeed, by definition, don't have a manageable likelihood, main reason why those models are opposed to probabilistic ones which instead can always count on powerful inferential tools.

Our focus lies in theoretical and methodological aspects, although there would exist a remarkable part of algorithmic implementation, and more precisely issues regarding mathematical foundation and asymptotic properties are carefully analyzed, inspired by an in-depth study of what is then our main bibliographic reference, that is [1] (see below), carrying out what follows: a rigorous formulation of the set-up for the ABC rejection algorithm, also to regain a transparent and general result of convergence as the ABC threshold goes to zero whereas the number $n$ of samples from the prior stays fixed; general technical proposals about distances leaning on OTT; weak assumptions which lead to lower bounds for small values of threshold and as $n$ goes to infinity, ultimately showing a reasonable possibility of lack of concentration in contrast to what is proposed in [1] itself.

**Main references.**


[1] E. Bernton, P. E. Jacob, M. Gerber, C. P. Robert. *Approximate Bayesian computation with the Wasserstein distance.* J. R. Statist. Soc. B (2019). Vol. 81, Issue 2, pp. 235–269.

[2] S. Chen, A. Mira, JP. Onnela. *Flexible model selection for mechanistic network models.* Cornell University, Statistics, Methodology (2018).

[3] R. Dutta, A. Mira, JP. Onnela. *Bayesian inference of spreading processes on networks.* The Royal Society (2018). Vol. 474, Issue 2215.

[4] A. Ebert, R. Dutta, K. Mengersen, A. Mira, F. Ruggeri, P. Wu. *Likelihood-free parameter estimation for dynamic queueing networks: case study of passenger flow in an international airport terminal.* Cornell University, Statistics, Methodology (2018).

[5] R. Goyal, JP. Onnela. *Framework for Converting Mechanistic Network Models to Probabilistic Models.* Cornell University, Physics, Physics and Society (2020).

[6] B. Jiang, TY. Wu, W. H. Wong. *Approximate Bayesian Computation with Kullback-Leibler Divergence as Data Discrepancy.* Conference on Artificial Intelligence and Statistics (2018).

[7] T. Lin, N. Ho, X. Chen, M. Cuturi, M. I. Jordan. *Revisiting Fixed Support Wasserstein Barycenter: Computational Hardness and Efficient Algorithms.* Cornell University, Computer Science, Computational Complexity (2020).

[8] H. D. Nguyen, J. Arbel, H. Lü, F. Forbes. *Approximate Bayesian computation via the energy statistic.* Cornell University, Statistics, Methodology (2019).

[9] V. M. Panaretos, Y. Zemel. *Statistical Aspects of Wasserstein Distances.* Annual Review of Statistics and Its Application (2019). Vol. 6, pp. 405-431.

[10] M. Park, W. Jitkrittum, D. Sejdinovic. *K2-ABC: Approximate Bayesian Computation with Kernel Embeddings.* Cornell University, Statistics, Machine Learning (2015).

[11] W. Rudin. *Real and Complex Analysis.* McGraw-Hill, 1986.

[12] S. A. Sisson, Y. Fan, M. A. Beaumont. *Handbook of Approximate Bayesian Computation.* Chapman & Hall/CRC, Handbooks of Modern Statistical Methods, 2019.

[13] B. K. Sriperumbudur, A. Gretton, K. Fukumizu, B. Schölkopf, G. R. G. Lanckriet. *Hilbert Space Embeddings and Metrics on Probability Measures.* Journal of Machine Learning Research (2010). Vol. 11, 1517-1561.

[14] A. Srivastava, W. Wu, S. Kurtek, E. Klassen, J. S. Marron. *Registration of Functional Data Using Fisher-Rao Metric.* Cornell University, Statistics, Machine Learning (2011).

[15] C. Villani. *Optimal Transport. Old and New.* Springer, 2009.

[16] V. Q. Vu, B. Yu, R. E. Kass. *Coverage-adjusted entropy estimation.* Statistics in Medicine (2007). Vol. 26, pp. 4039-4060.


**Supported missions.**

⋆ COST Short Term Scientific Mission on Statistics of Network Data Science (COSTNET STSM): *Approximate Bayesian Computation methods for Mechanistic Network Models* in collaboration with A. Mira and A. Ebert (Univ. of Italian Switzerland), L. Raynal (Univ. of Montpellier), C. Viscardi (Univ. of Florence) and F. Denti (Bicocca Univ. of Milan). 11 July - 7 November 2019. Web page: `here`.

**Attended talks.**

⋆ COSTNET final meeting 2020: *On the Mathematical Foundation of Approximate Bayesian Computation. A Robust Set for Estimating Mechanistic Network Models* in collaboration with D. Cassani (Univ. of Insubria) and A. Mira (Univ. of Italian Switzerland). 24 September 2020, online meeting (Zoom). Web page: `here`.

Paper and slides written in LaTeX available on `my web page`.