

## Statistical properties of deterministic and stochastic systems

The activity is focused on statistical properties of dynamical systems, in a wide perspective, encompassing both deterministic and stochastic evolutions.

In particular, the latest research projects concern systems whose phase space is unbounded, like random walks on infinite lattices, in the case of stochastic evolution, or Lorentz gases and area-preserving maps on the cylinder, in the deterministic setting. In the past decades many results have been achieved in understanding transport properties of such systems, pinpointing for instance where normal diffusion takes place and when anomalies arise.

From a general perspective such systems are not in equilibrium and their statistical properties are associated to infinite ergodic theory. In particular, since the fifties, occupation time statistics are known to be quite different from those exhibited by dynamics on a compact phase space, leading to (generalized) arc sine law, Sparre Andersen universality for persistence probabilities and Darling Kac theorem for residence in a bounded subset of the phase space.

Recent results on such themes encompass both stochastic evolution in a non-homogeneous lattice and deterministic systems mimicking the behaviour of polygonal billiard dynamics.

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## Quantum Transport and Thermodynamics

Since the industrial revolution, the transformation of heat into work has been at the center of technology. The earliest examples were steam engines, and current examples range from solar cells to nuclear power stations. The quest to understand the physics of this transformation led to the theory of thermodynamics.

The fast-paced development of nanoscale technologies is propelling thermodynamics into a new golden era. The miniaturization of devices, and in particular the emergence of new quantum technologies, is pushing the field of thermodynamics into new applied and fundamental challenges. Basic questions like the same definitions of work and heat in small systems, where quantum mechanics inevitably comes into play, have to be reconsidered with care and are vital to properly characterize the working of nanoscale thermal machines. The minimum temperature achievable in a finite time in small system is not only a fundamental question related to the third law of thermodynamics but also a practical question for cryogenic applications and in the initialization of qubits in a quantum computer. The development of protocols for heat flow control is the key point to evacuate heat and cool hot spots in nanodevices. These are just some of the several fundamental and practical challenges facing thermodynamics. More generally, connecting quantum thermodynamics, quantum information science and quantum physics is needed to develop novel, energy-efficient, sustainable quantum technologies.

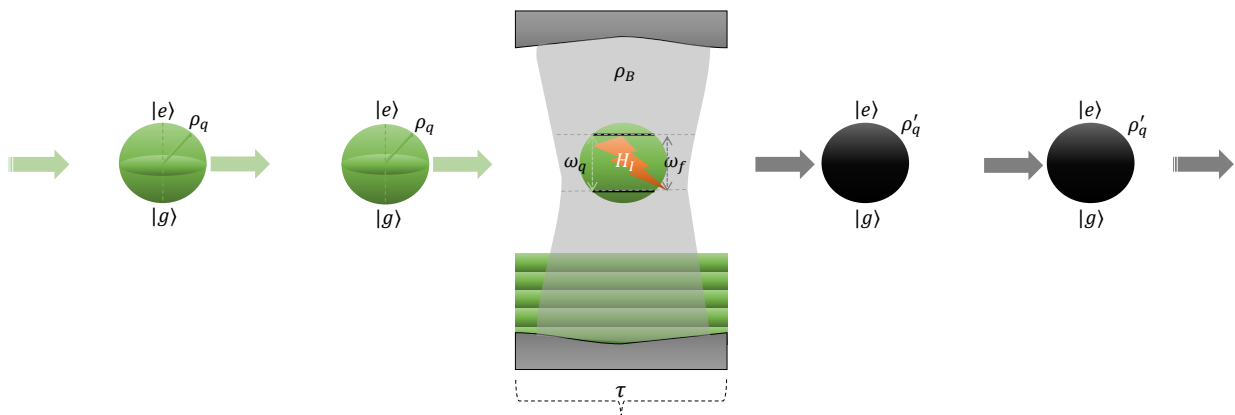
In recent years we have become increasingly interested in machines that convert heat into electrical power at a microscopic level. The perfect example of this are thermoelectric and photovoltaic devices. Nanotechnology has significantly advanced efforts in this direction, giving us unprecedented control of individual quantum particles. The questions of how this control can be used for new forms of heat to work conversion has started to be addressed in recent years. On the one hand, heat management at the nanoscale could dramatically reduce the energy cost of operating electronic devices and nanoscale scalable thermal engines could efficiently convert part of the waste heat into electricity. On the other hand, efficient thermoelectric Peltier cooling would underpin a wide range of quiet, long-lived refrigeration devices, with low environmental impact.

The study of cyclic (periodically driven) heat engines is challenging in the quantum domain, as it requires a deeper understanding of the thermodynamics of far from equilibrium quantum systems. In particular, the coupling, possibly strong, between quantum working medium and baths, can quite naturally induce non-Markovian effects, which may constitute a useful thermodynamic resource, for the construction and optimization of quantum thermal machines.

Any quantum machine needs energy to operate. Therefore, in parallel with developments on building nanodevices, topic of quantum batteries has emerged and flourished in the last few years. Quantum batteries are quantum mechanical system suitable to store energy in some excited states, to be released on demand. They represent a groundbreaking change of paradigm in the conception of energy manipulation and are opening new and

fascinating perspectives in this field. Indeed, the possibility of a quantum advantage, using quantum effects - coherence and entanglement - to increase the performance of a quantum battery has been the focus of several investigations.

Our main objective is to better understand the laws of nature. More particularly, we aim to better understand what heat and entropy mean for quantum objects, and to better understand how such objects thermalize. We also aim to understand what quantum machines can be capable of, and what the laws of physics do not allow. At the same time, more practical goals include using ideas from quantum transport and quantum thermodynamics to devise more efficient thermal rectifiers, quantum batteries, thermoelectrics, and cyclic heat engines.



*A pictorial view of a micromaser quantum battery [see Quantum Sci. Technol. 7, 04LT01 (2022)]*

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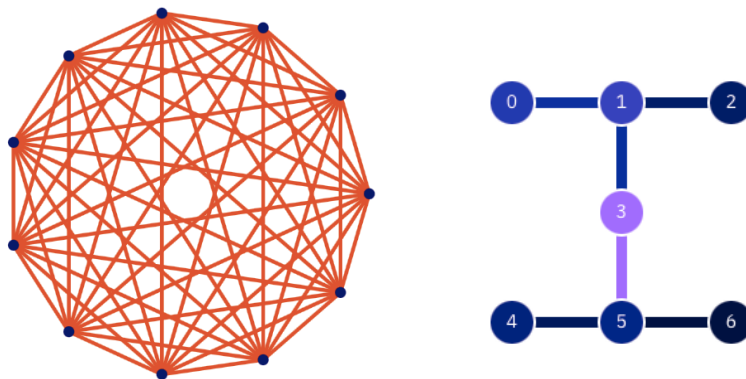
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## Simulating complex systems on actual quantum hardware

Quantum computers working with approximately 50–100 qubits could perform certain tasks beyond the capabilities of current classical supercomputers. As a general remark, quantum advantage can only be achieved if the precision of the quantum gates is sufficiently high and the executed quantum algorithm generates a sufficiently large amount of entanglement that can overcome classical simulation methods. Therefore, for quantum algorithms, multipartite (many-qubit) entanglement is the key resource to achieve exponential acceleration over classical computation. Unfortunately, existing noisy intermediate-scale quantum (NISQ) devices suffer from various noise sources such as noisy gates, coherent errors, and interactions with an uncontrolled environment. Noise limits the size of quantum circuits that can be reliably executed, so achieving quantum advantage in complex and practically relevant problems is still a formidable challenge.

Our purpose is to benchmark the progress of currently available quantum computers by simulating complex systems on different quantum hardware architectures. A non-exhaustive list of the problems we are currently studying includes the efficient generation of pseudo-random quantum states, hybrid quantum-classical variational approaches for state preparation, and energy transfer in quantum networks.



*Architectures of quantum processors used for the generation of pseudo-random states  
[see Entropy 25, 607 (2023)]*

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## Flocking active matter

Flocking – the collective motion exhibited by certain active matter (AM) systems – is a ubiquitous phenomenon, observed in a wide array of different living systems and on an even wider range of scales. While our knowledge of the bulk behavior in homogeneous systems, well described by the Toner & Tu (TT) theory, is now fairly complete, many questions remain unanswered when dealing with finite and or inhomogeneous systems. In particular we employ a combination of numerical and analytical tools to investigate

- The behavior of *confined collective motion*, such as flocks confined between parallel reflecting boundaries. Relevant questions include (i) the behavior of correlations functions and order parameter fluctuations when the rotational symmetry is locally broken at the boundaries as compared, for instance, to the one exhibited by systems driven by a homogeneous external field. (ii) The average behavior of the slow fields for the confined hydrodynamic theory.
- *Cohesive flocking through reinforcement learning*. It has been recently shown that flocking behaviour by local alignment may be learned by standard reinforcement learning algorithms in infinite systems. We are interested in generalizing this result to *finite flocks in open space*, where it has long been empirically known that local cohesion forces are also necessary.
- The *dynamics of heterogeneous flocks*. Many systems of biological interest are composed by a collection of heterogeneous self-propelled units. The simplest example is perhaps obtained mixing two populations, one capable of collective motion and a second one lacking collective alignment. We are interested in the collective dynamics of such mixtures in the high-density regime as the relative ratio of the two populations are varied, comparing numerical simulations of minimal models with experiments in hyper-confluent heterogeneous epithelial cells performed by a partner lab.

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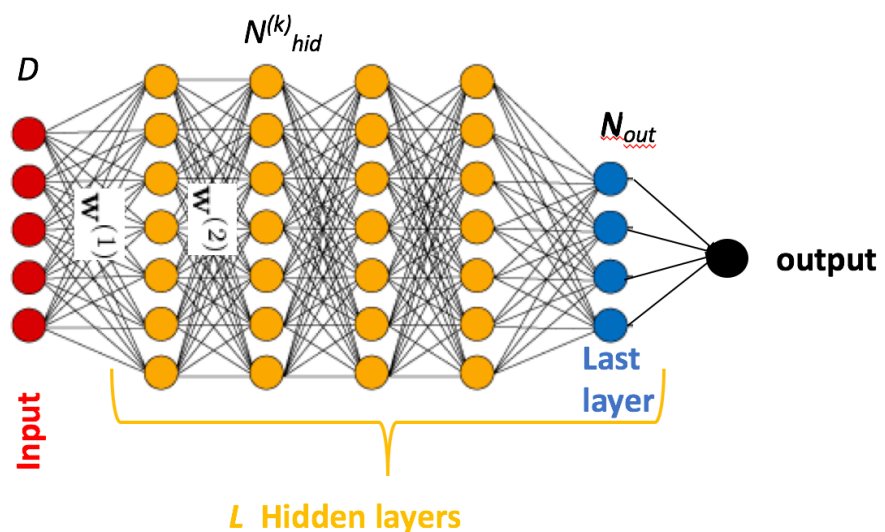
## Statistical Mechanics of deep learning

Modern deep neural networks (DNNs) represent a formidable challenge for theorists which are currently unable to fully understand the extraordinary success of state of the art DNNs at many tasks. Over the last years, it has been realized that huge simplifications arise in the infinite-width limit, where the number of units  $N$  in each hidden layer far exceeds the number  $P$  of training examples. This idealisation, however, departs from the reality of deep learning practice, where training sets are larger than the widths of the networks. We will investigate regression problems in finite-width deep networks in the so-called thermodynamic limit, where both  $N$  and  $P$  are large but their ratio finite, by means of tools from statistical mechanics and direct numerical experiments with both synthetic and real-world datasets. Key questions include

- The estimate of finite-width corrections in terms of the trained network generalization capabilities
- The pre-activation statistics of finite-width systems
- Under which conditions finite-width DNNs perform better than their infinite width counterparts.

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