RISM CONGRESS
ANALYSIS AND PDEs

On the occasion of Vicentiu Radulescu's 65th birthday

Vicentiu Radulescu

29 31 MAY 2023

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COMUNE DI VARESE

Abstracts of the talks

1. Henri Berestycki

The stability-compactness method and qualitative properties of nonlinear elliptic equations

In this talk I report on a series of works with Cole Graham on semi-linear elliptic equations with positive non-linearities, and Dirichlet boundary conditions. Solutions represent stationary states of reaction-diffusion equations. We focus in particular on uniqueness.

The main motivation is to study these equations in general unbounded domains, which exhibit remarkably rich behavior. Our method rests on decomposing the problem into a compact part and one for which a stability result can be derived and then to combine the two.

This approach has proved to be unexpectedly versatile and in fact encompasses past works on the subject such as the general moving plane method.

2. Lucio Boccardo

How much is positive the weak solution to a dirichle problem with positive datum?

In this talk I consider linear elliptic problem with a first order term (convection term or drift term).

Let Ω be a bounded, open subset of $\mathbb{R}^N$ and M be a bounded and measurable elliptic matrix; the vector field $E(x)$ belongs to $E \in (L^N(\Omega))^N$; $f(x)$, $g(x)$ belongs to $L^m(\Omega)$, $m \geq \frac{2N}{N+2}$.

Consider the Dirichlet problems ("formally" in duality)

$$\left| \begin{array}{l} u \in W_0^{1,2}(\Omega) : -\mathrm{div}(M(x)\nabla u) + u = -\mathrm{div}(u\,E(x)) + f(x); \\ \psi \in W_0^{1,2}(\Omega) : -\mathrm{div}(M(x)\nabla \psi) + \psi = E(x)\cdot\nabla\psi + g(x). \end{array} \right.$$

Existence and properties of solutions are studied in some recent papers. It is not trivial, but not too difficult to prove that the positivity of solutions of the above boundary value problems holds in the case of data f or g positive (that is, greater or equal than zero, but not identically zero).

It is possible to be more precise, indeed the solutions of the above boundary value problems can be zero at most in a zero measure set, if we assume that the data f or g are greater or equal than zero, but not identically zero. Then, some work in progress will be presented.

3. Francesca Crispo

On a parabolic p-Laplacian system of Burgers type

We are interested in the existence and regularity properties of solutions to the IBVP of a singular non-linear p-Laplacian system with a "convective" term, corresponding to a bounded initial data.

Assuming $\Omega \subset \mathbb{R}^n$ bounded, our main purpose is to prove the existence of solutions on $(0,\infty)\times\Omega$ with $L^r(\varepsilon,T;L^q(\Omega))$, $\varepsilon \geq 0$, integrability properties of the second spatial derivatives and of the time derivative.

As it happens in the scalar case, the main tool for existence are L^∞-estimates of solutions to approximating problems. For this task, we employ a duality argument, like in [2]. This gives a global L^∞ estimate of the solution, uniformly in $t \geq 0$, in agreement with known results for quasi-linear systems of Burgers type and with the classical p-Laplacian system.

Further, as $p < 2$, we study extinction properties for the solution too.

4. Joao-Paulo Dias

Magneto-optic effects in the motion of the director field of a nematic liquid crystal

This talk is based in a joint paper with P. Amorim and A. F. Martins, to appear. We study the motion of the director field of a nematic liquid crystal submitted to a magnetic field and to a laser beam.
The problem takes the form of a quasilinear wave equation in a single space variable, coupled to a Schrodinger equation.

We prove the existence of a global weak solution for the initial value problem using a viscous approximation and the compensated compactness method. We finish with some numerical computations to illustrate the results.

5. Josef Diblik

Linear delayed integro-differential equations and their uniform exponential stability

Uniform exponential stability of a linear delayed integro-differential vector equation is considered. The main result is of an explicit type, depending on all delays, and its proof is based on an a priori estimation of solutions, a Bohl-Perron type result, and utilization of the matrix measure.

A number of explicit stability conditions are obtained as corollaries, and the examples illustrate how they can improve previous stability results.

6. Roberta Filippucci

Multiplicity results for generalized quasilinear critical Schrödinger equations in RN

In this talk some recent results, obtained jointly with L. Baldelli, on generalized quasilinear Schrödinger equations in RN involving a nonlinearity which combines a power-type term at a critical level with a subcritical term, both with weights, will be discussed also in a symmetric setting.

The equation has been derived from models of several physical phenomena such as superfluid film in plasma physics as well as the self-channelling of a high-power ultra-short laser in matter.

7. Franco Flandoli

Remarks on Boussinesq eddy viscosity hypothesis from a stochastic analysis perspective

The additional dissipation provoked by small scale turbulence, conjectured by Joseph Boussinesq, so important for practical applications, remains debated. We have identified a rigorous mathematical scheme based on Stochastic Analysis which may help to clarify the validity of this hypothesis.
We discuss our present understanding of the topic in 2D and in 3D.

The research is supported by the European Commission Project NoisyFluid, n. 101053472

8. Genni Fragnelli

Degenerate wave equations with a drift term: new results on stabilization

In this talk we will present new results on the stabilization of the following problem

$$\begin{cases} u_{tt} - a(x)u_{xx} - b(x)u_x = 0, & (t,x) \in Q, \\ u(t,0) = 0, & t \in [0,+\infty), \\ \rho(u_t(t,1)) + \eta u_x(t,1) + \beta u(t,1) = 0, & t \in [0,+\infty) \\ u(0,x) = u_0(x), \quad u_t(0,x) = u_1(x), & x \in (0,1), \end{cases} \tag{1}$$

where $Q = (0,+\infty) \times (0,1)$, $f \in L^2_{\text{loc}}[0,+\infty)$, $a, b \in C^0[0,1]$, $a > 0$ on $(0,1]$ and $a(0) = 0$, η is a given function, β is a nonnegative constant and ρ is a given function.

Clearly the presence of the drift term leads us to use different spaces with respect to the usual ones; this fact, together with the nonlinear boundary condition, gives rise to several difficulties.
However, thanks to some suitable assumptions on the drift term, one can prove some estimates on the associated energy that are crucial to obtain a uniform exponential decay.

9. G.P. Galdi

Hopf Bifurcation and Resonance in a Fluid-Structure Problem

The flow of viscous fluid around structures is a fundamental problem that lies at the heart of the broad research area of Fluid-Solid Interaction. A main feature of this problem regards the study of the oscillations (vibrations) produced by the fluid on the structure and generated by Hopf bifurcation from the equilibrium (steady state) configuration.

In fact, they may lead either to useful and pleasant effects, like ringing wind chimes or Aeolian harps, or else destructive consequences, such as damage or even collapse of the structure. Of particular significance is the phenomenon of forced oscillation of suspension bridges. When the frequency of the oscillation induced by the fluid approaches the natural structural frequency of the bridge, a resonant phenomenon may occur that could culminate into structural failure.

In this presentation we will provide a mathematical analysis of flow-induced oscillations due to Hopf bifurcation, on classical models proposed by the current engineering literature. However, this analysis shows that, at least for the commonly adopted models, a dramatic structural failure cannot just be ascribed to resonance effects.

10. Olivier Goubet

Non-existence results for an eigenvalue problem involving Lipschitzian nonlinearities with non-positive primitive

We discuss the existence and non-existence of non zero classical solutions to equations as $\Delta u= \sin u$ on bounded domains of $\mathbb{R}^D$, with homogeneous Dirichlet boundary conditions. We give some consequence on the global attractor for damped sine-Gordon equations. This is a joint work with B. Ricceri (Catania).

11. Chao Ji

Multiplicity and concentration results for the magnetic Schrodinger equations with exponential critical growth in R2

In this talk, we are concerned with the nonlinear magnetic Schrodinger equations with exponential critical growth in R2. Under a local assumption and a global assumption on the potential V respectively, we show multiplicity and concentration of solutions.

Moreover, we establish the multiplicity of multi-bump solutions for the nonlinear magnetic Schrodinger equation with the deepening potential well. This talk is based on joint works with Pietro d'Avenia and Vicentiu D. Radulescu.

12. Paolo Marcellini

Regularity for elliptic equations under general growth conditions

We give some existence and interior regularity results for weak solutions of elliptic equations in divergence form of the type

$$\sum_{i=1}^{n} \frac{\partial}{\partial x_i} a^i\left(x, u\left(x\right), Du\left(x\right)\right) = b\left(x, u\left(x\right), Du\left(x\right)\right) ,$$

in an open set $\Omega \subset \mathbb{R}^n$, $n \geq 2$. The vector field $\left(a^i\left(x, s, \xi\right)\right)_{i=1,2,\ldots,n}$ satisfies some *general growth conditions* with respect to the gradient variable $\xi \in \mathbb{R}^n$, the so-called $p, q-$*growth conditions*.

The novelties with respect to the mathematical literature on this topic are the general growth conditions and the explicit dependence of the differential equation on u, other than on its gradient Du and on the x variable.

The seminar will also examine the problem of the existence and multiplicity of weak solutions of the Dirichlet problems associated with the above elliptic differential equation, in the context of p; q-growth.

13. Enzo Mitidieri

Recent results on superharmonic functions in a half space

We prove a representation formula for superharmonic functions on the half space RN+ := RN−1×]0,+∞[. As a result, we derive various comparison principles, qualitative properties of solutions of integral and differential inequalities, as well as related Liouville theorems.

14. Giovanni Molica Bisci

Nonlocal critical problems with jumping nonlinearities

In this talk we deal with nonlocal critical growth elliptic problem driven by the fractional Laplacian in presence of jumping nonlinearities. Using variational and topological methods and applying a new linking theorem, we prove the existence of a nontrivial solution for the problem under consideration.

These existence results can be seen as the nonlocal counterpart of the ones obtained in the context of the Laplacian equations. In the nonlocal framework the arguments used in the classical setting have to be refined. Indeed the presence of the fractional Laplacian operator gives rise to some additional difficulties, that we are able to overcome proving new regularity results for weak solutions of nonlocal problems, which are of independent interest.
This is a joint paper with Kanishka Perera, Raffaella Servadei and Caterina Sportelli.

15. Dimitri Mugnai

On some mixed operators

We deal with two classes of Dirichlet boundary value problems driven by mixed local-nonlocal operators. In the first case we consider the sum of the p-Laplacian and of the fractional p-Laplacian, showing a result à la Brezis-Oswald. In the second part we present a new mixed operator of interest in peridynamics, where the nonlocal operator acts only in a proper subset of the spatial domain, showing some qualitative and quantitative results.

These are joint results with S. Biagi and E. Vecchi (in the first case) and with F. Cluni, V. Gusella, E. Proietti Lippi and P. Pucci (in the second case).

16. Nikolaos S Papageorgiou

Singular Double Phase problems

We consider a Dirichlet problem with two parameters driven by the double phase differential operator and with a reaction which has the competing effects of a singular term and a superlinear perturbation. We prove an existence and multiplicity result which is global in the first parameter when the second is Jarge.

17. Giulio Romani

Nonlocal planar Schrödinger-Poisson systems in the fractional Sobolev limiting case

In this talk I will present some recent results concerning existence and symmetry for a nonlocal planar Schrödinger-Poisson system driven by the $s-$fractional $p-$Laplacian with $p=\frac2s$. Since the dimension two is the limiting case for the embedding of the fractional Sobolev space $W^{s,p}(\mathbb{R}^2)$, one can deal with exponential growths.
We prove the existence of solutions by means of a variational approximating procedure for an auxiliary Choquard equation equation in which we uniformly approximate the sign-changing logarithmic kernel of the Poisson equation. Qualitative properties of solutions such as symmetry and decay are also established by exploiting a suitable moving planes technique.
These results have been obtained in collaboration with D.Cassani (Università dell'Insubria, RISM) and Z.Liu (Wuhan University).

18. Delia Schiera

Spectral optimization for weighted anisotropic problems with Robin conditions

I will present some recent results obtained in collaboration with B. Pellacci and G. Pisante on a weighted eigenvalue problem with anisotropic diffusion in bounded Lipschitz domains under Robin boundary conditions.

We prove the existence of two positive eigenvalues $\lambda^\pm$ respectively associated with a positive and a negative eigenfunction, and we analyze the minimization of $\lambda^\pm$ with respect to the sign-changing weight, showing that the optimal weights are of bang-bang type, namely piece-wise constant functions, each one taking only two values.

As a consequence, the problem is equivalent to the minimization with respect to the subsets of the domain satisfying a volume constraint. Then, we completely solve the optimization problem in one dimension, in the case of homogeneous Dirichlet or Neumann conditions, showing new phenomena induced by the presence of the anisotropic diffusion. The optimization problem for λ^+ naturally arises in the study of the optimal spatial arrangement of resources for a species to survive in an heterogeneous habitat.

19. Enzo Vitillaro

Three evolution problems modelling the interaction between acoustic waves and non–locally reacting surfaces

We deal with three evolution problems arising in the physical modelling of acoustic phenomena of small amplitude in a homogeneous, ideal, nonviscous and compressible fluid, bounded by a surface of extended reaction. The problems are posed in a bounded and simply connected domain Ω of $\mathbb{R}^3$ with boundary $\Gamma = \partial\Omega$ of class $C^{r,1}$ with $r = 2, 3, \ldots, \infty$, where $\Gamma = \Gamma_0 \cup \Gamma_1, \overline{\Gamma_0} \cap \overline{\Gamma_1} = \emptyset, \Gamma_1$ being nonempty and connected.

The first problem arises from modelling the phenomenon in an Eulerian framework, so the fields characterizing the problem are incremental pressure, velocity and boundary deformation. The second

one arises from modelling the phenomenon in a Lagrangian framework, so the fields appearing are the displacement and the boundary deformation. The third one is the well–known wave equation with acoustic boundary conditions.

In the talk we shall give well–posedness results for the Cauchy problems related to the first two models and give the exact relations between the three problems, which are not exactly equivalent.

20. Runzhang Xu

Global existence and blow up of solutions for pseudo-parabolic equation with singular potential

In this talk, we like to report a study in the initial boundary value problem of pseudo-parabolic equation with singular potential, in order to classify the initial data for the global existence, finite time blowup and longtime decay of the solution.

The whole study is conducted by considering three cases according to initial energy: low initial energy case, critical initial energy case and high initial energy case. For the low initial energy case and critical initial energy case the sufficient initial conditions of global existence, long time decay and finite time blowup are given to show a sharp-like condition.
Also two different strategies are applied to estimate the upper bounds of the blowup time for the negative initial energy blowup and positive initial energy blowup respectively. And for the high initial energy case, the finite time blowup is proved.

21. Weiping Yan

Stabilizability for a quasilinear Klein-Gordon-wave system with variable coefficients

In this talk, we concerns with the stabilizability for a quasilinear with variable coefficients in whole space.

22. Binlin Zhang

On a critical and singular Kirchhoff-type problem

In this talk, we discuss a three-dimensional Kirchhoff-type problem involving critical and singular nonlinearities.
By combining variational methods with some delicate decomposition techniques, we obtain the existence of two positive solutions in the case of low perturbations of the singular nonlinearity, namely for small values of the parameter. Here we point out that our decomposition techniques could be applied to more elliptic equations with critical growth.
This is a joint work with C. Lei and V. Radulescu.

23. Jian Zhang

Double phase problems with competing potentials: concentration and multiplication of ground states

In this talk, we will introduce some recent results about the perturbed double phase problem with lack of compactness and competing potentials. More precisely, applying topological and variational tools

from Nehari manifold analysis and Ljusternik Schnirelmann category theory, we study the existence of positive ground state solutions and the relation between the number of positive solutions and the topology of the set where the nonlinear potential attains its global minimum and the nonlinear potential attains its global maximum.
Moreover, we determine two concrete sets related to the potentials as the concentration positions and we describe the concentration of ground state solutions. Finally, the asymptotic convergence and the exponential decay of positive solutions are also explored.
This talk is based on joint works with Vicentiu D. Radulescu and Wen Zhang.

24. Jianjun Zhang

Another look at planar Schrodinger-Newton systems

In this talk, we focus on the existence of positive solutions to a planar Schrodinger-Newton system with subcritical or critical growth. A new variational approach is established and enables us to study such problem in the Sobolev space as usual.

The analysis developed in this paper also allows to investigate the relation between a Schrodinger-Newton system of Riesz-type and a Schrodinger-Poisson system of logarithmic-type. Furthermore, this new approach can provide a new look at the planar Schrodinger-Newton system and may have some potential applications in various related problems.

25. Xuexiu Zhong

On Bartsch-Jeanjean-Soave's open problem and a global branch approach to the study of normalized solution problem

Sirakov's open problem[Comm.Math.Phys.,2007] is focused on the best range of the parameters where the positive solution exists for an Bose-Einstein Condensation system with prescribed frequencies. Parallelly, Bartsch,Jeanjean and Soave [J.Math.Pures Appl.,2016] proposed an open problem focusing on the best range of the parameters guaranteeing the existence of positive solution for the Bose-Einstein Condensation system (with prescribed masses).

In this talk, I will introduce a global branch approach developed (with Bartsch and Zou) in [Math. Ann. 2021] and give an answer to Bartsch-Jeanjean-Soave's open problem. In particular, in the preprint [arXiv:2112.05869], we (with Jeanjean and Zhang) further develop a global branch approach without eigenvalues, and successfully applying it to give a unified way to study normalized solutions of Schrodinger equations, which are either mass subcritical, mass critical or mass supercritical.

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Thanks for you participation

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Supported by

A.D. 1308 unipg Dipartimento di Giurisprudenza

Accademia delle Scienze dell'Umbria

con il contributo di Patrizia Pucci